

NPV



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Infinity

Now we can use what we've learned about summation, infinity, fractions, and present value to derive formulas from accounting & finance –

$$\sum_{t=1}^{\infty} \frac{A}{(1+r)^t} = \frac{A}{(1+r)^1} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \dots$$



Review



Earlier, we introduced a formula –

$$\sum_{t=1}^{\infty} X^t = \frac{X}{1-X} \quad 0 < X < 1$$

This is close but not exactly the same as our expression

$$\sum_{t=1}^{\infty} \frac{A}{(1+r)^t}$$



Quick Edit

$$\frac{\frac{1}{1+r}}{1 - \frac{1}{1+r}}$$

We can restate our expression so it fits the formula

$$\sum_{t=1}^{\infty} \frac{A}{(1+r)^t} = A \left[\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \right] = A \left[\sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t \right]$$



The Brackets

$$\left[\sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t \right] = \frac{1/(1+r)}{1 - 1/(1+r)}$$

$$\frac{1/(1+r)}{(1+r)/ (1+r) - 1/(1+r)}$$

$$\frac{1/(1+r)}{r/(1+r)} = \frac{1}{r}$$

$$\frac{\frac{1}{(1+r)}}{\frac{(1+r)}{(1+r)} - \frac{1}{(1+r)}}$$

$$\frac{\frac{1}{(1+r)}}{\frac{r}{(1+r)}} = \frac{1}{r}$$



In Conclusion

$(1+r)$

From before –

$$\sum_{t=1}^{\infty} \frac{A}{(1+r)^t} = A \left[\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \right] = A \left[\sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t \right] = \frac{A}{r}$$

This is the “perpetuity formula” from finance.

It tells us the present value of an infinite series of equal amounts.

These can be costs (e.g. we pay the bank \$2500 every month)

These can be revenues (e.g. our investments earn 80 million baht every year)



Quick Example

We invest 750 million Baht TODAY.

Our investment returns an income of 80 Million baht every year forever...



There is a 10% interest rate...

The present value of the returns equals $\frac{80 \text{ Million}}{.10} = 800 \text{ Million baht}$

The NET PRESENT VALUE of this investment equals:

$$\text{NPV} = -750 \text{ Million} + 800 \text{ Million} = 50 \text{ Million baht}$$



Takeaways

We derived the perpetuity formula

We introduced the concept of using NPV as an investment guide



- we invest 750 m
- money going out -750 m
- what do we get back
 - investment gives 80 m
 - * this is *perpetuity*
- interest rate is 0.1 (10%)

NET PRESENT VALUE (NPV) – SIMPLE EXPLANATION

EXAMPLE

- Initial investment today: 750 million baht
- Annual cash flow received forever: 80 million baht
- Discount rate (required return): 10%

WHAT DOES THE 10% INTEREST RATE MEAN?

The 10% is NOT applied directly to the 750 million investment cost.

It is the required rate of return (also called the discount rate).

Think of it as:

"If I had money today, I could invest it somewhere else and earn 10% per year."

Therefore, when evaluating future cash flows, we use 10% to determine what those future payments are worth today.

UNDERSTANDING THE PRESENT VALUE

The investment pays 80 million baht every year forever.

The question is:

"How much money would I need today so that earning 10% per year would generate 80 million baht annually?"

Let that amount be X.

$$10\% \times X = 80 \text{ million}$$

$$0.10 \times X = 80$$

$$X = 80 / 0.10$$

$$X = 800 \text{ million}$$

This means:

If you had 800 million baht today and invested it at 10% per year:

$$800 \text{ million} \times 10\% = 80 \text{ million}$$

You could withdraw 80 million every year and leave the original 800 million untouched.

Therefore:

Receiving 80 million baht every year forever is worth 800 million baht today.

PERPETUITY FORMULA

For a perpetuity:

$$PV = C / r$$

Where:

PV = Present Value

C = Annual Cash Flow

r = Discount Rate

In this example:

$$PV = 80 / 0.10$$

$$PV = 800 \text{ million baht}$$

CALCULATING NPV

NPV = Present Value of Future Cash Flows - Initial Investment

$$NPV = 800 - 750$$

$$NPV = 50 \text{ million baht}$$

INTERPRETATION

- Cost today: 750 million baht

- Value of future cash flows: 800 million baht
- Extra value created: 50 million baht

Because NPV is positive (+50 million baht), the investment is considered worthwhile.

SIMPLE ANALOGY

Imagine a bank account that pays 10% interest.

If you deposit 1,000 baht:

$$1,000 \times 10\% = 100 \text{ baht per year}$$

So receiving 100 baht every year forever is equivalent to owning 1,000 baht today.

Likewise:

80 million per year forever at a 10% discount rate is equivalent to 800 million baht today.

KEY TAKEAWAY

The 10% rate is not applied to the 750 million investment cost.

Instead, it is the rate used to convert future cash flows into today's value.

The 80 million annual cash flow and the 10% discount rate together imply a present value of 800 million baht.

Since 800 million > 750 million, the investment has a positive NPV of 50 million

baht.